EEN 360
MODERN
CONTROL

SPRING 2008
TEST 2
90 MINUTES

NDU
ECCE DEPARTMENT
S. C. ELMURR

14/55

NAME

NOTE1: OPEN BOOK, OPEN NOTES, CLOSED OLD TESTS AND SOLUTIONS.
NOTE2: SHOW ALL WORK IN ORDER TO RECEIVE FULL CREDIT.

1. 55 Pts. The open- loop transfer function of a single-loop negative feedback system is

$$GH(s) = \frac{K(s+8)}{s(s+3)(s+5)(s+7)(s+15)}$$

This system is called conditionally stable because it is stable for only a range of the gain K as follows: $k_1 < K < k_2$. Using the Routh-Hurwitz criteria and the root-locus method, determine the range of the gain for which the system is stable. Sketch the root locus for $-\infty < K < \infty$.

Hint:(the breakaway points are -11.6, -9.34, -6.4, -4.01, -1.12 and $k_2 = 1010$

1) 25, 2) 10, 3) 10

① Poles : $s = 0$, $s = -3$, $s = -5$, $s = -7$, $s = -15$

Zeros : $s = -8$, 4 at ∞

② # of branches = 5

③ Asymptotes = $|n - m| = |5 - 1| = 4$

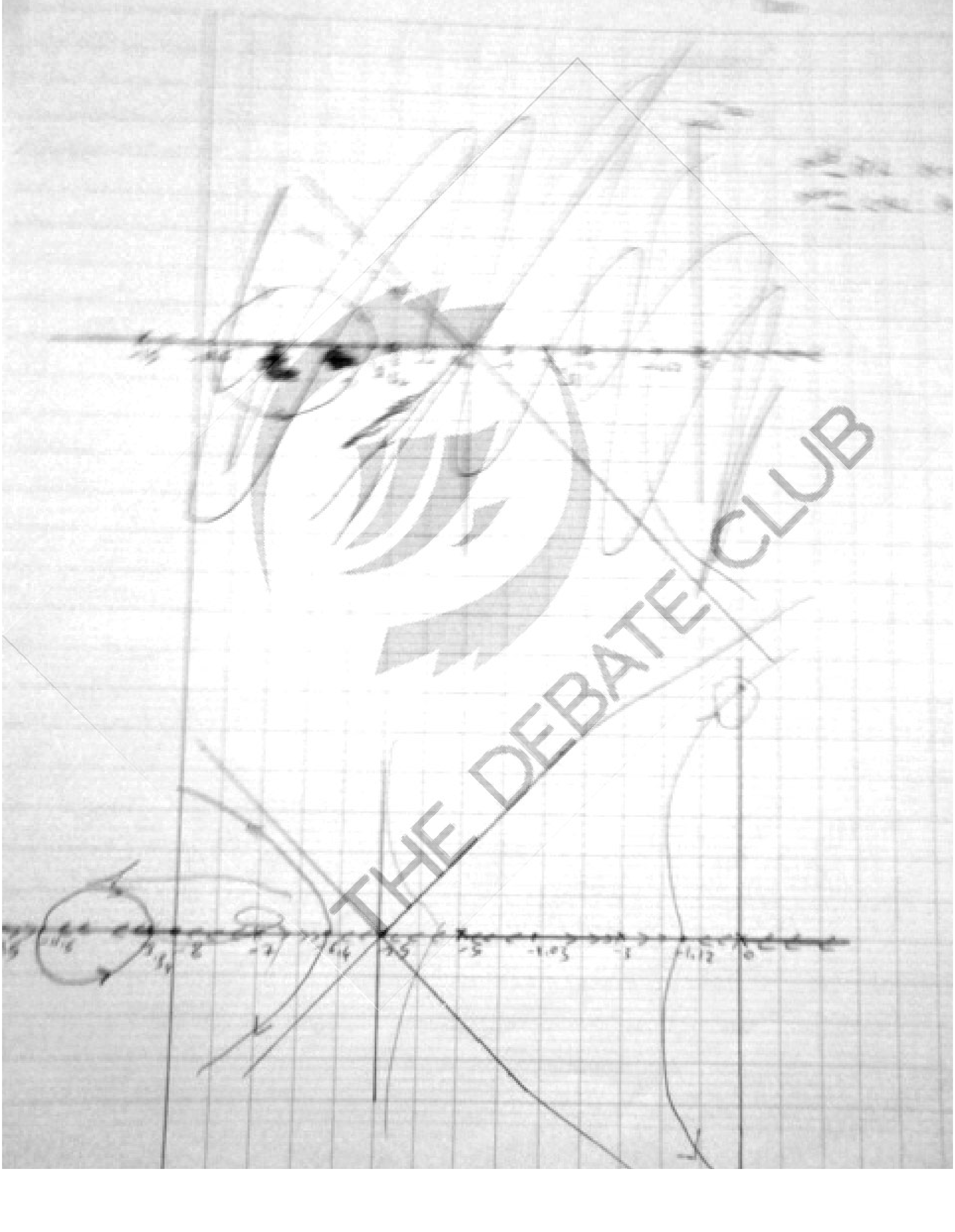
④ Centroid $= \dfrac{\Sigma P - \Sigma Z}{n - m} = \dfrac{(0 - 3 - 5 - 7 - 15) - (-8)}{4}$

$= -5.5$

$i = 0, 1, 2, 3$

⑤ RL $\dfrac{(2i+1) \times 180}{|n - m|} \Rightarrow 45;\ 135;\ 225;\ 315$

CRL $\dfrac{(2i)\,180}{|n - m|} \Rightarrow 0;\ 90;\ 180;\ 270$



2. 25. Pts. For the system described below, find the transformation $x(t) = Q\bar{x}(t)$ so that the state equations are transformed into observability canonical form (OCF) if possible.

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad C = [1\ 0\ 1]$$

$$|sI - A| = \begin{bmatrix} s & -2 & 0 \\ -1 & s-2 & 0 \\ 1 & -1 & s-1 \end{bmatrix}$$

$$= s(s-2)(s-1) + (2)(-s+1)$$

$$= (s^2 - 2s)(s-1) - 2s + 2$$

$$= s^3 - 2s^2 - s^2 + 2s - 2s + 2$$

$$= s^3 - 3s^2 + 2$$

$a_0 = 2 \quad a_1 = 0 \quad a_2 = -3$

$$\bar{A} = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\bar{C} = [0\ 0\ 1]$$

$$Q = (MV)^{-1}$$

$$M = \begin{bmatrix} 0 & -3 & 1 \\ -3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Q = (MV)^{-1}$$

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 1 \\ 2 & 7 & 1 \end{bmatrix} \qquad Q = \left(\begin{bmatrix} 0 & -3 & 1 \\ -3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 1 \\ 2 & 7 & 1 \end{bmatrix} \right) = ?$$

5

-6

$$CA = [1\ 0\ 1] \times \begin{bmatrix} 0 & 2 & 0 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} = [-1\ 3\ 1]$$

$1\times3 \qquad 3\times3$

$$CA^2 = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 2 & 0 \end{bmatrix} [-1\ 3\ 1] = [2\ 7\ 1]$$

3. 20. Pts. A system has the following differential equation:

$$\dot{x} = \begin{bmatrix}-1 & 0\\ 2 & -3\end{bmatrix}x + \begin{bmatrix}0\\ 1\end{bmatrix}r(t).$$

Determine $\Phi(s)$ and $\Phi(t)$ for the system.

$$A = \begin{bmatrix}-1 & 0\\ 2 & -3\end{bmatrix}$$

$$\phi(s) = [sI - A]^{-1}$$

$$(sI - A) = \begin{bmatrix}s+1 & 0\\ -2 & s+3\end{bmatrix}$$

$$|(sI - A)| = (s+1)(s+3)$$

$\text{Adj}\,[sI - A]$

$$\Rightarrow ① \begin{bmatrix}s+3 & 2\\ 0 & s+1\end{bmatrix} \Rightarrow ② \text{ Transpose } \begin{bmatrix}s+3 & 0\\ 2 & s+1\end{bmatrix}$$

$$\text{Adj}\,[sI - A] = \begin{bmatrix}s+3 & 0\\ 2 & s+1\end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+1)(s+3)}\begin{bmatrix}s+3 & 0\\ 2 & s+1\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{s+1} & 0\\ \frac{2}{(s+1)(s+3)} & \frac{1}{(s+3)}\end{bmatrix}$$

$$\frac{A}{s+1} + \frac{B}{s+3} = \frac{2}{(s+1)(s+3)}$$

$$A = \frac{2}{s+3}\Big|_{s=-1} = 1$$

$$B = \frac{2}{s+1}\Big|_{s=-3} = -1$$

$$\Rightarrow [sI - A]^{-1} = \begin{bmatrix}\frac{1}{s+1} & 0\\ \frac{1}{(s+1)} - \frac{1}{(s+3)} & \frac{1}{s+3}\end{bmatrix}$$

$$\phi(t) = \mathcal{L}^{-1}\left([sI - A]^{-1}\right)$$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{1}{s+1} - \frac{1}{s+3} & \frac{1}{s+3} \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t} & 1 \\ e^{-t} - e^{-3t} & e^{-3t} \end{bmatrix}$$